19002719

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Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, OCTOBER 2019

First Semester

Faculty of Science

Branch I (a) : Mathematics

MT 01C05—COMPLEX ANALYSIS

(2012—2018 Admissions)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Define conformal mappings. Give an example.
2. Does $f(z) = e^z$ conformal in the whole plane? Justify.
3. State Cauchy's theorem. Compute $\int_\gamma x\, dz$ where γ is the directed line segment from 0 to $1 + i$.
4. State Liouville's theorem. Does the function $f(z) = \sin z$ bounded ? Justify.
5. Does the integral of an exact differential over any cycle is zero? Justify.
6. State Maximum principle.
7. Define Harmonic function. Give an example.
8. State Poisson's formula.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Find the fixed points of the linear transformation $w = \dfrac{3z-4}{z-1}$.
10. State the symmetry principle. Prove that every reflection carries circles into circles.
11. State and prove Cauchy's Integral formula.
12. Compute $\int_{|z|=1} \dfrac{\sin z}{z}\, dz$.

Turn over

13. Show that a function which is analytic in the whole plane and has a nonessential singularity at ∞ reduces to a polynomial.
14. State and prove fundamental theorem of algebra.
15. Find the poles and residues of the function $f(z)=\frac{1}{\sin z}$.
16. How many roots of the equation $z^4-6z+3=0$ have their modulus between 1 and 2 ?

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.
18. Prove that a function which is analytic in the whole plane and satisfies the inequality $|f(z)|<|z|^n$ for some n and all sufficiently large $|z|$ reduces to a polynomial.
19. Show that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.
20. State and prove Taylor's theorem.
21. How many roots of the equation $z^4 + 8z^3 + 3z^2 + 8z + 3 = 0$ lie in the right half plane ?
22. Evaluate the integral $\int_0^{\infty} \frac{\cos x}{x^2+a^2}\,dx$, a real.

(3 × 5 = 15)